Distributed Estimation-based Formation Control with Orientation Alignment

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Abstract. This paper proposes a distributed estimation algorithm to carry out distributed formation control with orientation alignment in stochastic multi-agent systems(MAS). The main objective is to address inconsistent agent orientations of the individual agents and achieves the formation goal without knowing global reference frame nor initial misalignments between the agents' local frames. Existing research on orientation alignment has been limited by its reliance on the inter-agent communication, often vulnerable to data transmission delay, packet drop, etc. To circumvent these issues, the proposed algorithm solely exploits local sensing information, enabling agents to achieve orientation alignment based on their individual observation data with respect to their own local frame without needing communication. Then, as long as the MAS network observability condition is satisfied, all the agents' orientations can achieve the alignment, as well as the formation converges to the desired formation. The practical effectiveness of the proposed algorithm is demonstrated with numerical simulations with multi-rotor formation flight.

Keywords: Distributed estimation, orientation alignment, distributed formation control

1 Introduction

Distributed formation control has recently attracted a considerable amount of research interest, with distance-constrained formation control taking a prominent role [1]. This assumes that each agent is able to measure the relative positions of their neighbors, and the target formation is specified by inter-agent distances and orientations. However, due to misalignments between local frames of multi-agent systems (MAS), cooperative operation for a target formation can be difficult without a global reference frame. Some existing methods propose a consensus-style orientation control law for aligning the orientations of local reference frames [2]. In this setting, agents achieve the formation goal and orientation alignment without requiring initial orientation information. However, these approaches require the exchange of locally sensed measurement information between agents through inter-agent communication, leading to potential risks such as data transmission delay and packet drop, etc. In this work, we introduce a distributed estimation algorithm to develop a formation control law based on orientation alignment, which eliminates data transmission between agents and reaches the desired formation goal without an initial common reference frame. The proposed estimation strategy allows each agent to estimate and control the orientation through the local observation of formation maneuver, thereby ensuring a common sense of orientation for all agents. This guarantees the convergence of the target formation if the MAS network observability condition is satisfied.

2 Problem Formulation

In this section, we model the distributed estimation algorithm for a multi-agent system (MAS) consisting of N homogeneous agents. The stochastic linear time-invariant dynamics of the entire MAS is described by:

$$\begin{aligned} X(k+1) &= \tilde{A}X(k) + \tilde{B}U(k) + \tilde{w}(k) \\ \tilde{A} &= \begin{bmatrix} A_x & 0 \\ 0 & A_\theta \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B_x & 0 \\ 0 & B_\theta \end{bmatrix} \end{aligned} \tag{1}$$

where $X = [x, \theta]^T$ and $U = [u, w]^T$ represent the state and the control input of all agents, respectively. $x = [x_1, \ldots, x_N]^T$ and $\theta = [\theta_1, \ldots, \theta_N]^T$ represent the position and the orientation of all agents, respectively, and k represents the time step. The disturbance in the dynamics, $\tilde{\omega}$, is modeled by Gaussian noise with zero mean and covariance matrix $\Theta \succ 0$. Using additional notation, the position and the position control input of agent i with respect to its local reference frame are expressed as $x_i^i(k)$ and $u_i^i(k)$, respectively. We define the relative position between agents i and j with respect to ith agent's local reference frame as follows:

$$x_{ji}^i(k) = x_j^i(k) - x_i^i(k), \tag{2}$$

and the relative orientation between agents i and j is defined as:

$$\theta_{ji}(k) := PV(\theta_j(k) - \theta_i(k)), \tag{3}$$

where $PV(\theta_j(k) - \theta_i(k)) := [(\theta_j(k) - \theta_i(k) + \pi) \bmod 2\pi] - \pi$ is a protocol that modifies orientation angles to lie within an interval of range less than π. It is assumed that $\theta_{ji}(k)$ is not available to agent i without the global reference frame Σ^g or data transmission through inter-agent communication methods or protocols.

The ith agent is assumed to be able to measure the noisy positions of its neighboring agents with respect to its local reference frame. The measurement of the ith agent is given by the following:

$$Z_i(k) = H_i(x^i(k) + v_i(k)), \tag{4}$$

where $x^i = [x_i^i, \ldots, x_N^i]^T$ is the position vector of the MAS with respect to ith agent's local reference frame, H_i is the measurement matrix for the ith agent, and v_i is the measurement noise vector modeled by Gaussian noise with zero mean and covariance matrix $\Xi_i \succ 0$

2.1 Formation Control Problem

Consider the consensus-based formation control input with respect to the global reference frame denoted by Σ^g:

$$\begin{aligned} u_i(k) &= F_x \sum_{j \in N_i} (x_{ji}(k) - x_{ji,des}) \\ &= (L_i \otimes F_x)(x(k) - x_{des}), \end{aligned} \tag{5}$$

where F_x is the position control gain, and L_i is the ith row of the Laplacian matrix. Using the Laplacian matrix L, the position control input of the entire MAS can be rewritten as

$$u(k) = (L \otimes F_x)(x(k) - x_{des}). \tag{6}$$

Note that due to the absence of a common orientation reference, agent i maintains its local reference frame, with its origin at the state $x_i(k)$ and its orientation angle $\theta_i(k)$ relative to Σ^g.

2.2 Consensus-based Orientation Alignment Protocol

In this section, we attempt to align the orientations of all agents to a common sense of reference frame Σ^a using a consensus-based orientation alignment protocol as follows:

$$\begin{aligned} w_i(k) &= F_\theta \sum_{j \in N_i} \theta_{ji}(k) \\ &= L_i F_\theta \theta(k), \end{aligned} \tag{7}$$

where w_i is the angular velocity of agent i. Using the Laplacian matrix L, the angular velocity of the entire MAS can be rewritten as

$$w(k) = L F_\theta \theta(k). \tag{8}$$

It is assumed that the initial orientations are within an interval with a range less than π. Note that $\theta_{ji}(k)$ cannot be directly observed by individual agents, and thus an orientation estimation strategy, which will be discussed in Section 3, is required.

3 Algorithm Development

3.1 Distributed State Estimator Design

Given the distributed estimator embedded in each agent, the state estimate of the MAS from the ith agent's perspective is denoted as:

$$\hat{X}^i(k) := \mathbb{E}[X(k) | Z_{i,(0:k)}],$$

and the corresponding estimation error covariance is represented as:

$$\Sigma^i(k) := \mathbb{E}[e^i(k)e^i(k)^T | Z_{i,(0:k)}],$$

where $e^i(k)$ is the estimation error. Additionally, the predicted state estimation and its estimation error covariance at the next time step are denoted as:

$$\hat{X}^{i-}(k+1) := \mathbb{E}[X(k+1)|Z_{i,(0:k)}],$$

$$\Sigma^{i-}(k+1) := \mathbb{E}[e^{i-}(k+1)e^i(k+1)^T | Z_{i,(0:k)}].$$

Acquiring the measurement $Z_i(k)$ at each time step, the state estimate of the MAS from the ith agent's perspective is recursively updated by the estimator. Based on the proposed state and orientation estimator, each agent can implement the estimation-based feedback control law as follows:

$$u_i^i(k) = \tilde{R}(\theta^i(k))(L_i \otimes F_x)(\hat{x}^i(k) - x_{des}), \tag{9}$$

$$w_i(k) = LF_\theta \hat{\theta}^i(k), \tag{10}$$

where $\tilde{R}(\theta^i) = blkdg(R(\theta_i), \ldots, R(\theta_N))$ represents a block diagonal-matrix with Rotation matrix. Along with (1), (9), (10), the nonlinear predicted state estimate is expressed as follows:

$$\begin{aligned}\hat{x}^{i-}(k+1) &:= f_x(\hat{x}^i(k), \hat{\theta}^i(k)) \\ &= A_x\hat{x}^i(k) + B(L \otimes F_x)\left(\hat{x}^i(k) - \tilde{R}^{-1}(\hat{\theta}^i(k))x_{des}\right),\end{aligned} \tag{11}$$

$$\begin{aligned}\hat{\theta}^{i-}(k+1) &:= f_\theta(\hat{x}^i(k), \hat{\theta}^i(k)) \\ &= (A_\theta + BF_\theta L)\hat{\theta}^i(k),\end{aligned} \tag{12}$$

where $f(x, \theta)$ is the nonlinear function of the predicted state estimate from the perspective of the ith agent. In (11), (12), the ith agent attempts to predict the entire MAS formation maneuver from the perspective of the ith agent with respect to its local reference frame. By calculating the Jacobian matrix:

$$J_i(k) = \begin{bmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial \theta} \\ \frac{\partial f_\theta}{\partial x} & \frac{\partial f_\theta}{\partial \theta} \end{bmatrix}_{\hat{x}^i, \hat{\theta}^i},$$

the predicted error covariance can be calculated as:

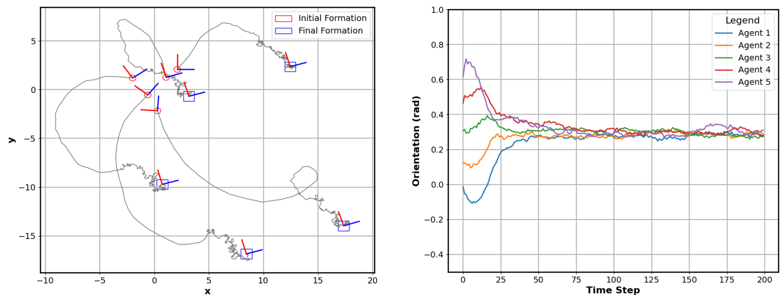
$$\Sigma^{i-}(k+1) = J_i(k)\Sigma^i(k)J_i^T(k) + \Sigma_{\tilde{\omega}}. \tag{13}$$

The updated state and the error covariance are calculated from the predicted state and the new measurement at time step $k+1$:

$$\hat{X}^i(k+1) = \hat{X}^{i-}(k+1) + K_i(k+1)\left(Z_i(k+1) - H_i\hat{X}^{i-}(k+1)\right), \tag{14}$$

$$\Sigma^i(k+1) = (I - K_i(k+1)H_i)\Sigma^{i-}(k+1), \tag{15}$$

where K_i is the kalman gain, and H_i filters out the MAS state entries of non-neighboring agents of the ith agent.



(a) (b)

Fig. 1: (a) Traces of positions, with the measurement noise and the disturbance characterized by variances of $0.1m^2$; (b) trajectories of orientations.

4 Numerical Simulation

The proposed algorithm is demonstrated in a formation flight to arrange 5 homogeneous agents' orientations into a common sense of orientation, ensuring the target formation geometry without the need for inter-agent communication. To emulate sensor noise and the disturbance for our simulation, we infuse synthetic zero-mean Gaussian noise with $0.1m^2$ variance into the measurements and the dynamics. Figure 1 shows that the agents achieve orientation alignment and reach the target formation by solely relying on observation data.

5 Conclusion

In this research, we proposed a distributed estimation strategy that enables formation control and orientation alignment in stochastic MAS without inter-agent communication. By relying on locally sensed measurements for each agent, the proposed algorithm overcomes orientation inconsistencies in misaligned local frames, ensuring target formation convergence.

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References

1. Lin Z., Francis B., Maggiore M. Necessary and sufficient graphical conditions for formation control of unicycles. *IEEE Transactions on Automatic Control*,2005.
2. Oh K.K., Ahn H.S. Formation control and network localization via orientation alignment. *IEEE Transactions on Automatic Control*,2013.