Optimal Thrust Estimation Considering Actual Thrust Fitting of Quadrotors

Joonhyun Kim[1] and Kyunam Kim[1,2]

[1] Dept. of Intelligent Robotics, Sungkyunkwan University, Suwon, South Korea
[2] Dept. of Mechanical Engineering, Sungkyunkwan University, Suwon, South Korea
`Corresponding author (kyunam.kim@skku.edu)`,

Abstract. As the technology of unmanned aerial vehicles (UAVs) advances, their use is increasing in various fields such as defense and delivery. In particular, the demand for delivery systems using quadrotors has surged. To meet this demand, companies are exploring payload transportation using UAVs. However, quadrotor cargo delivery faces challenges when encountering unpredictable forces, such as wind. Therefore, solving the issue of stabilizing payloads under unstable conditions becomes a crucial aspect of ensuring the reliable operation of quadrotor delivery systems. This study aims to correct the thrust control input by applying a second-degree polynomial to the actual thrust, allowing the actual thrust to be estimated.

Keywords: Quadrotor, Thrust compensation, Thrust Estimation,

1 Introduction

The growth of large e-commerce platforms has led to an increased demand for fast and efficient delivery systems. Consumers desire quicker deliveries, and companies are seeking efficient transportation methods to meet this demand. Consequently, for efficient cargo transportation, the use of aerial mobility for payload transport, particularly utilizing unmanned aerial vehicles (UAVs), has been explored [1,2].

Recently, studies have been conducted on transporting payloads with multiple quadrotors. This method has demonstrated improved efficiency, reliability, and stability in transportation compared to a single quadrotor [3]. A geometric nonlinear control system for the dynamic model of multiple quadrotors jointly carrying a payload has been studied [4]. Additionally, a control algorithm for multiple quadrotors carrying a payload has been proposed [5]. However, when external disturbances such as wind or gusts occur, the load on the quadrotor fluctuates, potentially destabilizing its flight. As the first step in addressing this issue, accurately estimating the real-time thrust generated by the quadrotor is essential.

In this paper, we aimed to find a linear relationship by using a second-order polynomial regression to linearize the nonlinear relationship between the control input PWM and thrust. We applied a technique that converts the quadrotor's control input into a Pulse Width Modulation (PWM) signal to estimate thrust. PWM is a method of controlling motor output by adjusting the power delivered to the motor through pulse width conversion of the control input. The motor drives the propeller, which generates thrust, allowing the quadrotor's thrust to be adjusted through control inputs.

Since there is a nonlinear relationship between control input and thrust, we used a second-order polynomial to map the control input to the actual thrust. Through this approach, the quadrotor can adjust motor speed based on the corrected input, enabling it to accurately manage and estimate the applied thrust.

2 Related Work

This section reviews previous studies related to thrust correction and battery voltage compensation in quadrotors. The basic method for quadrotor thrust correction involves performing static thrust tests and approximating the thrust-to-speed curve [6]. Instead of the standard quadratic model, a polynomial equation with three coefficients was used to approximate the thrust mapping. In another approach [7], the rotor thrust curve for motor commands was identified using the quadratic model, and a linear term was added to account for the effects of battery voltage depletion. Similarly, in [8] and [10], methods using a load cell mounted on a nano quadrotor were used to adjust the thrust through battery voltage compensation by applying random PWM signals in a static state. In [9], the study addressed quadrotor control in situations of battery voltage drop, integrating dynamic models and controllers through PWM signals, while using fractional-order sliding mode control (FSMC) for attitude control. However, these approaches present the issue that the thrust values do not match from the perspective of actual thrust during quadrotor flight. Additionally, they cannot consistently maintain the relationship between the control input and actual thrust during flight. This makes them unsuitable for real-time accurate thrust estimation in quadrotors. We develop a thrust correction algorithm by fitting the control input to the actual thrust using a second-order regression, thereby making the relationship between the control input and actual thrust. By linearizing this relationship, the control system can be designed using more intuitive and simple equations. This reduces the complexity of the control algorithm, making its implementation and tuning easy.

3 Background

In this section, we provide the necessary background knowledge for estimating the thrust of the quadrotor.

3.1 Cascaded PID Controller Based Thrust PWM

In Fig. 2, the trajectory generator provides position information as a setpoint, and the cascaded PID position controller generates control signals based on the

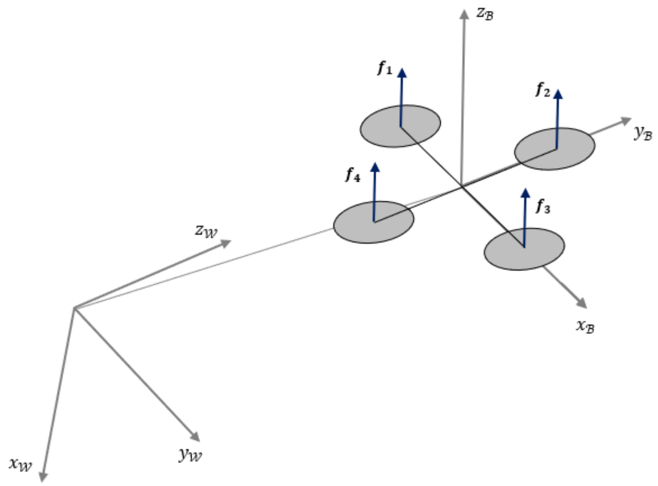

Fig. 1. Quadrotor Dynamic Model

difference between the current position and the target position. These control signals, derived from PID gain parameters, are transmitted as PWM signals. Similarly, the attitude controller adjusts the quadrotor's attitude based on signals received from the higher-level controller. The output vector generated in this block is defined as $\mathbf{u} = [u_1, u_2, u_3, u_4]^T$, where u_1 represents thrust f, and u_2,u_3, and u_4 correspond to moments that control rotational motion. These values are converted into PWM signals, which then distribute power to each motor based on these signals. Eq. (1) describes how the quadrotor's direction or attitude is adjusted according to the control input vector $\mathbf{u}$, where h_1, h_2, h_3, h_4 are the PWM signals delivered to the motors. By selectively increasing or decreasing $\mathbf{u}$, the thrust and attitude of the quadrotor are controlled during flight.

$$\begin{aligned} h_1 &= u_1' - u_2 + u_3 + u_4 \\ h_2 &= u_1 - u_2 - u_3 - u_4 \\ h_3 &= u_1 + u_2 - u_3 + u_4 \\ h_4 &= u_1 + u_2 + u_3 - u_4 \end{aligned} \tag{1}$$

3.2 Real Thrust Quadratic polynomial mapping

We account for the nonlinearity between the control input u_1 and the actual thrust f_a, and apply a second-order polynomial regression based on the least squares method to model the relationship between u_1 and f_a. By doing so, we compute a new control input u_1', which linearly transforms the relationship between the control input and the actual thrust.

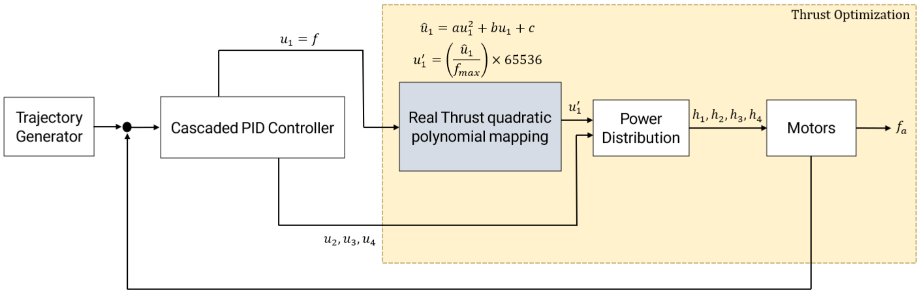

Fig. 2. Diagram of the proposed control framework. Based on the input set-points, the position controller from the Cascaded PID Controller generates the control input u_1, corresponding to the thrust. This input is then adjusted to $\hat{u}_1$ through quadratic polynomial mapping of the actual thrust and ultimately transformed into the final control input u_1'.

4 Approach

This section explains the method of linearly mapping the PWM of the control input u_1, which represents thrust f, to the actual thrust f_a. The actual performance of the quadrotor motors is experimentally measured to improve the accuracy of the model. By inputting various PWM values to the motors and measuring the generated thrust proportional to the weight of the quadrotor, the PWM can be adjusted to estimate the thrust according to the actual load of the quadrotor.

4.1 Real Thrust Quadratic polynomial mapping

In thrust control for a quadrotor, an important factor is how precisely the computed thrust from the control input aligns with the real thrust. To address this, we adopted a simplified model to represent the relationship between the control input (PWM) and the actual thrust. Specifically, we assumed a linear relationship between PWM and the motor's rotational speed (RPM) and a quadratic relationship between RPM and thrust. Based on these assumptions, we ultimately fit a quadratic model between PWM and thrust to create a relatively simple model. Here, the relationship between the control input u_1 and actual thrust f_a is nonlinear. However, our goal was to find a thrust that has a linear relationship with u_1 through quadratic fitting, and this thrust is defined as f_d. To achieve this, we performed a second-order polynomial regression based on the least squares method to model the relationship between the control input u_1 and actual thrust f_a, thus calculating a new control input u_1' that linearly transforms the relationship between the control input and actual thrust.

One of the critical factors in controlling the thrust of a quadrotor is how accurately the thrust calculated from the control input can predict the actual thrust. Ideally, the desired thrust f_d corresponding to the control input u_1 should exhibit a linear relationship, but in reality, the actual thrust f_a is nonlinear. Considering this nonlinearity, a second-order polynomial regression based on the least squares method is performed to model the relationship between the control input u_1 and the actual thrust f_a. Using this, we compute a new control input u_1' to linearly transform the relationship between the control input and the actual thrust.

$$\hat{u}_1 = au_1^2 + bu_1 + c \tag{2}$$

$$u_1' = (\frac{\hat{u}_1}{f_{max}}) \times 65536 \tag{3}$$

where u_1' is the transformed control input. $\hat{u}_1$ is the control input corresponding to the actual thrust. f_{max} represents the maximum thrust that the quadrotor can produce. 65536 is the maximum value that can be used in the 16bit PWM signal of the quadrotor. In Eq. (2), the goal is to find the constants a,b, and c such that the data points closely fit the actual thrust curve. In Eq. (3), the transformed control input $\hat{u}_1$ is normalized by dividing it by the maximum thrust f_{max}, and this ratio is multiplied by 65536 to convert the control signal u_1' into a PWM signal that can be sent to the motor. In other words, the control signal is scaled to fit within the 16bit PWM value range.

5 Experiment

The quadrotor used in this study is a Bitcraze Crazyflie 2.1 [11], which is equipped with an STM32 microcontroller unit that performs low-level flight control and estimation tasks. The Crazyflie 2.1 originally offers a thrust of 60 grams, but by applying the Thrust Upgrade bundle [12] in place of the original propellers and motors, 5 extra grams of thrust were gained from each motor and propeller, resulting in a combined thrust of 80 grams.

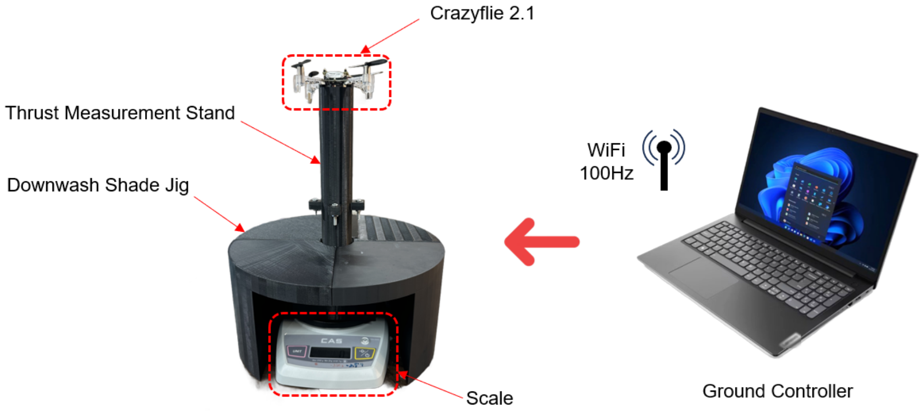

Fig. 3. Thrust Measurement Stand.

5.1 Thrust Measurement Stand Setup

To accurately measure thrust, the following considerations are necessary: i) it is important to verify how much actual thrust is generated in relation to the PWM input. ii) the weight reduction observed on the scale may not directly correspond to the thrust generated due to the ground effect caused by the rotating propellers, which can influence the scale reading.

Considering these issues, we designed a thrust measurement stand, as shown in Fig. 3, to accurately measure thrust and conduct optimization experiments. The thrust measurement stand was created using 3D printing based on the designed model. Instead of using a load cell, we fixed a Crazyflie 2.1 to the thrust measurement stand and placed it on a scale to check the actual thrust. A shade jig was fabricated to allow the downwash from the propeller to flow away, preventing it from affecting the scale. The experiment was conducted by sending control input u_1 to the Crazyflie 2.1 at a 100Hz cycle via a ground controller to control the thrust.

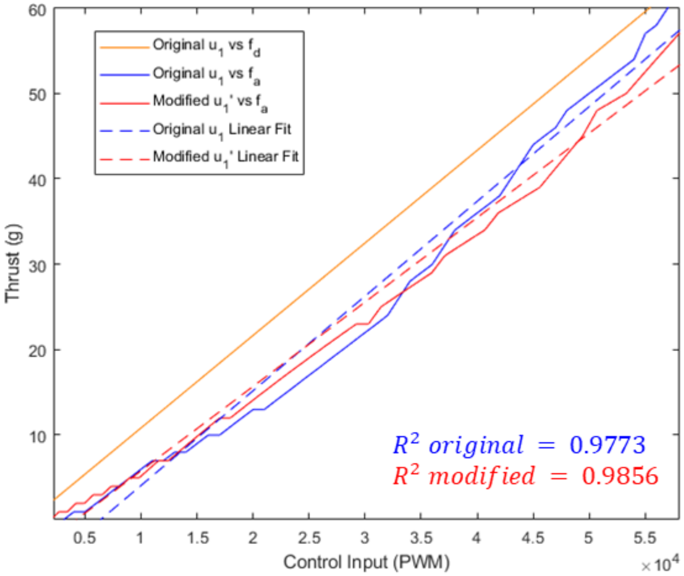

Fig. 4. Comparison between Control input(PWM) and Thrust(g) i) Original u_1 vs f_d, ii) Original u_1 vs f_a, iii) Modified u_1' vs f_a, iv) Original u_1 Linear Fit, v) Modified u_1' Linear Fit. Comparison. Comparison of R-Squared between the original model and the modified model.

5.2 Thrust Correction Experiment

In Fig. 4, f_d represents the ideal thrust value that can be obtained from the control input u_1. To compare the actual thrust f_a, before and after thrust correction using a second-order polynomial, we tested the Crazyflie 2.1 in Fig. 3. We conducted multiple experiments by increasing the control input u_1 from a PWM value of 1000 to 60000, which is approximately 91.5% of the control input. During the process, we manually recorded the actual thrust f_a on a scale with a one-second interval between each increase in the control input. Using the obtained f_a values and the control input u_1, we applied second-order polynomial regression with Eq. (2) to find the constants a, b, and c in order to linearly approximate the nonlinear relationship between the control input and the actual thrust. Additionally, we measured the actual maximum thrust f_{max}, which was 71 grams. By applying this value to Eq. (3), we calculated the corrected control input signal u_1 required to achieve the desired actual thrust f_a. The thrust correction coefficients obtained through the second-order linear regression are $a = 6.40308 \times 10^{-9}$, $b = 0.000742171$ and $c = -0.9899922188$. The new variables are implemented into the quadrotor firmware, and the thrust alignment is compared with the existing model in Fig. 4.

The comparison method evaluates model performance by comparing the R-squared values of the plots between the original $u_1 - f_a$ model and the post-firmware $u_1' - f_a$ model to determine how well the input values predict the output values. The original $u_1 - f_a$ model without thrust correction variables has an R-squared value of 0.9773, while the modified $u_1' - f_a$ model has an R-squared value of 0.9856, indicating that the modified model fits the data for the relationship between control input and actual thrust better. The $u_1' - f_a$ model should theoretically be linear, but it still has a quadratic tendency. This issue arises because the interaction between the motor and the propeller is proportional to the square of the motor's rotational speed (ω), meaning that even when the control input is approximated linearly, there can still be a discrepancy with the actual thrust. As shown in Fig. 4, the nonlinearity becomes more pronounced when the motor's rotational speed is too low or too high.

In conclusion, while approximating the relationship between control input and thrust using a quadratic polynomial offers simplicity and computational efficiency, it cannot fully capture the nonlinear characteristics that occur in real-world conditions, making additional nonlinear corrections necessary.

6 Conclusion and Future Work

This paper's goal is to find a linear relationship by using a second-order polynomial regression to linearize the inherently nonlinear relationship between the control input PWM and thrust. This approach establishes an environment that enables accurate estimation of the thrust system, allowing the quadrotor to estimate actual thrust and respond effectively to subtle thrust changes even in complex flight environments. Such thrust correction enhances thrust estimation accuracy by maintaining consistent thrust under various conditions, thereby improving the overall efficiency and stability of the system. However, there were limitations in achieving a perfectly linear relationship between the control input and thrust.

Therefore, future research will focus on modeling the relationship between control input and actual thrust more accurately by using higher-order polynomial regression. Additionally, a system will be established that allows a quadrotor to log its load by generating thrust logs corresponding to PWM signals, and the thrust generated during actual flights will be analyzed using a motion capture system. This will enable the implementation of more precise thrust estimation for quadrotors.

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